Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Friday 20 May 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. Use the standard results for $\sum_{r=1}^{n} r$ and for $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r^2 - 3) = \frac{n}{4}(n+a)(n+b)(n+c)$$

where a, b and c are integers to be found.

(4)

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Question 1 continued

Q1

(Total 4 marks)

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2. A parabola P has cartesian equation $y^2 = 28x$. The point S is the focus of the parabola P.

(a) Write down the coordinates of the point S.

(1)

Points A and B lie on the parabola P. The line AB is parallel to the directrix of P and cuts the x-axis at the midpoint of OS, where O is the origin.

(b) Find the exact area of triangle ABS.

(4)

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Question 2 continued

Q2

(Total 5 marks)

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3.

$$f(x) = x^2 + \frac{3}{x} - 1, \qquad x < 0$$

The only real root, α, of the equation $f(x) = 0$ lies in the interval $[-2, -1]$.

(a) Taking -1.5 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 2 decimal places.
(5)

(b) Show that your answer to part (a) gives α correct to 2 decimal places.
(2)

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Question 3 continued

Q3

(Total 7 marks)

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4. Given that

$$\mathbf{A} = \begin{pmatrix} k & 3 \\ -1 & k+2 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

(a) show that $\det(\mathbf{A}) > 0$ for all real values of k,

(3)

(b) find $\mathbf{A}^{-1}$ in terms of k.

(2)

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Question 4 continued

Q4

(Total 5 marks)

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5.

$$2z + z^* = \frac{3 + 4\mathrm{i}}{7 + \mathrm{i}}$$

Find z, giving your answer in the form $a + b\mathrm{i}$, where a and b are real constants. You must show all your working.

(5)

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Question 5 continued

Q5

(Total 5 marks)

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6. The rectangular hyperbola H has equation $xy = 25$

(a) Verify that, for $t \neq 0$, the point $P\left(5t, \dfrac{5}{t}\right)$ is a general point on H.

(1)

The point A on H has parameter $t = \dfrac{1}{2}$

(b) Show that the normal to H at the point A has equation

$$8y - 2x - 75 = 0$$

(5)

This normal at A meets H again at the point B.

(c) Find the coordinates of B.

(4)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total 10 marks)

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7.

$$\mathbf{P} = \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation U represented by the matrix **P**.

(3)

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line with equation $y = x$

(b) Write down the matrix **Q**.

(1)

Given that the transformation V followed by the transformation U is the transformation T, which is represented by the matrix **R**,

(c) find the matrix **R**.

(2)

(d) Show that there is a value of k for which the transformation T maps each point on the straight line $y = kx$ onto itself, and state the value of k.

(4)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 10 marks)

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8.

$$f(z) = z^4 + 6z^3 + 76z^2 + az + b$$

where a and b are real constants.

Given that $-3 + 8i$ is a complex root of the equation $f(z) = 0$

(a) write down another complex root of this equation.
(1)

(b) Hence, or otherwise, find the other roots of the equation $f(z) = 0$
(6)

(c) Show on a single Argand diagram all four roots of the equation $f(z) = 0$
(2)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 9 marks)

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9. The quadratic equation

$$2x^2 + 4x - 3 = 0$$

has roots α and β.

Without solving the quadratic equation,

(a) find the exact value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

(5)

(b) Find a quadratic equation which has roots $(\alpha^2 + \beta)$ and $(\beta^2 + \alpha)$, giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers.

(4)

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Question 9 continued

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Question 9 continued

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Question 9 continued

(Total 9 marks)

Q9

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10. (i) A sequence of positive numbers is defined by

$$u_1 = 5$$
$$u_{n+1} = 3u_n + 2, \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 2 \times (3)^n - 1$$

(5)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{(3 + 2n)}{3^n}$$

(6)

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Question 10 continued

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Question 10 continued

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Question 10 continued

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Question 10 continued

Q10

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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